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ON A CLASS OF SINGULAR DIFFERENTIAL OPERATORS

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In this note, the author will report some results for a class of non-Fuchsian singular hyperbolic operators including

$$L = (t\partial_t)^2-\Delta_x+a(t,x)(t\partial_t)+\langle b(t,x),\partial_x\rangle+c(t,x).$$

1. CLASS OF OPERATORS.

Let $(t,x)=(t,x_1,\dots,x_n)\in\mathbb{R}_t\times\mathbb{R}^n_x$ and let

$$P = (t\partial_t)^m+\sum_{\substack{j+|\alpha|\leq m\\ j<m}} a_{j,\alpha}(t,x)(t\partial_t)^j\partial_x^\alpha,$$

where $m\in\{1,2,\dots\}$, $\partial_t=\partial/\partial t$, $\partial_x=(\partial/\partial x_1,\dots,\partial/\partial x_n)$, $\alpha=(\alpha_1,\dots,\alpha_n)\in\{0,1,2,\dots\}^n$, $|\alpha|=\alpha_1+\dots+\alpha_n$, $\partial_x^\alpha=(\partial/\partial x_1)^{\alpha_1}\dots(\partial/\partial x_n)^{\alpha_n}$, and the coefficients $a_{j,\alpha}(t,x)$ ($j+|\alpha|\leq m$ and $j<m$) are C^∞ functions defined in an open neighborhood U of $(0,0)$ in $\mathbb{R}_t\times\mathbb{R}^n_x$. Our main assumption on P is as follows:

(A) All the roots of

$$\lambda^m+\sum_{\substack{j+|\alpha|=m\\ j<m}} a_{j,\alpha}(t,x)\lambda^j\xi^\alpha = 0$$

are <u>real</u>, <u>simple</u> and <u>non-zero</u> for any $(t,x,\xi)\in U\times(\mathbb{R}^n_\xi\setminus\{0\})$.

Remark 1. Note that P is not of Fuchsian type in t.

Remark 2. Recall that the typical model of Fuchsian hyperbolic operators in t is the following:

$$R = (t\partial_t)^m + \sum_{\substack{j+|\alpha|\leq m \\ j<m}} a_{j,\alpha}(t,x)(t\partial_t)^j(t^k\partial_x)^\alpha,$$

where $(t^k\partial_x)^\alpha = (t^k\partial/\partial x_1)^{\alpha_1}\dots(t^k\partial/\partial x_n)^{\alpha_n} (= t^{k|\alpha|}\partial_x^\alpha)$ and the following conditions are imposed on R.

(B-1) $k \in \mathbf{Z}$ and $k>0$.

(B-2) All the roots of

$$\lambda^m + \sum_{\substack{j+|\alpha|=m \\ j<m}} a_{j,\alpha}(t,x)\lambda^j\xi^\alpha = 0$$

are real and simple for any $(t,x,\xi) \in U \times (\mathbb{R}^n_\xi \setminus \{0\})$.

(B-3) All the roots of

$$\rho^m + \sum_{j<m} a_{j,0}(0,0)\rho^j = 0$$

are non-integers.

2. SOME RESULTS.

Here, we want to consider the following problems (I)~(V) for P.

(I) Is Pu=f solvable in C^∞ ?

(II) Is Pu=f solvable in $\mathcal{D}'$?

(III) Can we construct a parametrix ?

(IV) Does the uniqueness of the C^∞-solution hold ?

(V) Is every solution $u \in \mathcal{D}'(t>0)$ of Pu=0 extendable to $\{t \leq 0\}$ as a distribution ?

In order to make clear our situation, we present the following table which compares the results for P with those for Fuchsian operators R.

	non-Fuchsian case		Fuchsian case	
operator	P under (A)		R under (B-1)~(B-3)	
Problem (I)	Yes	[T,S]	Yes	[T]
Problem (II)	Yes	[T]	Yes	[B-L-P,T]
Problem (III)	Yes	[S]	Yes	[B-L-P-T]
Problem (IV)	No	[M]	Yes	[T,R,U]
Problem (I)	Conjecture No*)		Yes	[P-T]

In the above table, we quoted names by their initials: T=Tahara, S=Serra, B=Bove, L=Lewis, P=Parenti, M=Mandai, R=Roberts and U=Uryu.

As to *): the case n=1 is already proved; but the case $n \geq 2$ is still open (up to the date Nov. 14, 1988).

3. CONCLUSION.

By the results in Section 2, we may assert that our class of non-Fuchsian operators has an interesting nature and therefore it is worthy to investigate it.